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INFLUENCE LINES FOR MOMENT AND SHEAR IN A CONTINUOUS BEAM

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INFLUENCE LINES FOR MOMENT AND SHEAR IN A CONTINUOUS BEAM

Anthony Hoadley,[1] M. ASCE

Conjugate Beams Useful in Analyzing Unusual Conditions

The influence line is a valuable tool to use in studying the effect of moving loads on beams or trusses or in studying the effect of placing a heavy concentrated load at different positions on a building frame. Many problems can be handled by using the influence line tables appearing in the "Steel Construction Manual" of the American Institute of Steel Construction or in "Influence Line Tables" by Griot-Larsch to mention two sources. In some instances the problem may involve span stiffness ratios, panel length or end support conditions which are not covered in the available tables so that an independent solution becomes necessary. Influence lines for beams rigidly attached to columns or beams in which the moment of inertia varies within the span cannot be readily found from these tables. The following numerical examples are presented to show how influence lines equations can be found for continuous beams by use of the conjugate beam. Equations are also presented giving the moment and shear influence lines in a continuous beam of three spans. Each span has a different stiffness factor K ($K = I/1$) and the ends of the continuous beam are simply supported. The use of the conjugate beam for determining influence lines has been suggested in "Theory of Statically Indeterminate Structures" by Fife and Wilbur and in unpublished notes developed by Neil Welden, A.M., ASCE.

The influence line for reaction at A, shear at B, or moment at C, in a continuous beam is identical with the appropriately chosen deflection curve for the beam. For reaction the deflection curve is that obtained when the support at A is moved a unit distance vertically. To obtain the shear influence line the beam is cut at B and the cut ends subjected to a relative vertical displacement of unity, while sections of the beam to the right and left of point B must maintain the same slopes. For the moment influence line at C the beam is assumed to be hinged at C and moments applied which will cause the tangents for the sections to the right and left of C to undergo a relative rotation of one radian. That the influence line is identical with the appropriate deflection diagram can be proved by using the theorem developed by the Englishman James C. Maxwell—better known for his work in electricity and physics than for his important work in structural theory—showing the reciprocal nature of the deflection at any two points in an elastic system. The German engineer, Prof. Mueller Breslau was the first man to show the identity of the influence diagram and deflection curve.

The conjugate beam is a useful figment of the imagination so devised that the moment at a point in the conjugate beam is equal to the deflection at the corresponding point in the real beam and the shear at a point in the conjugate

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beam is equal to the slope of the corresponding point in the real beam. The conjugate beam is as long as the real beam and is loaded with the M/EI diagram of the real beam. The conjugate beam must be articulated so that shears and moments will be consistent with known slopes and deflection at points of support in the real beam. If the end of a beam is simply supported it has slope but no deflection at that point. The corresponding point in the conjugate beam must carry shear but no moment, consequently it will also be simply supported. When the real beam is fixed at the end it has neither slope nor deflection. The conjugate has a free end at this point as it must carry neither shear nor moment. At an interior support the real beam has slope but no deflection. This calls for a hinge in the conjugate which will carry shear but not moment.

Moment Influence Lines for Beam on Supports

The conjugate beam method of computing influence lines will be illustrated by finding the influence lines for moment and shear at a point P located 40 ft. to the right of support 1 of the beam of Fig. 1a. The conjugate of this beam is shown in Fig. 1b. The slope of the real beam changes suddenly at P, consequently a support must be provided at P in the conjugate beam to make possible its corresponding change in shear.

The moment influence line is obtained by applying a moment which will cause a relative rotation of one radian at P, finding the M/EI diagram for the real beam caused by this applied moment and then finding the moment curve in the conjugate beam when loaded with the M/EI diagram of the real beam. That the moment curve in span 2 due to an internal moment M_p acting at P is the trapezoid of Fig. 2 can be shown as follows. The moment M_p causes moments M_{cd} and M_{dc} at the ends c and d of span 2. Taking section cd of the continuous beam as a free body the only external forces acting are the end moments and the shears which they produce. M_{cd} produces a triangular moment diagram with its maximum at c and M_{dc} a moment diagram with its maximum at d. Combining these moment diagrams results in a trapezoidal moment diagram for span 2. The following conventions will be used:

1) Bending moment is positive when the top fibre of a beam is in compression.
2) Fixation moments used in moment distribution are positive when the applied moment tends to turn the joint at the end of a beam clockwise.
3) A positive M/EI diagram is treated as an upward load.
4) Positive moment in conjugate beam gives positive influence values.
5) x is always measured from the far end of any span toward P.

The moments at the supports of the real beam caused by the applied moment can be found by moment distribution or by the slope deflection method if the fixation moments F_{12}, F_{21} at the 1 and 2 ends of span 2 are known. These values are:

$$F_{12} = \frac{E\,I_2}{L_2}\,(4-6k)^{*} = EK_2\,(4-2.40) = 1.60\ EK$$

$$F_{21} = \frac{E\,I_2}{L_2}\,(6k-2)^{*} = EK_2\,(2.40-2) = 0.40\ EK$$

*Derivations given in appendix

The method of moment distribution was used here. The results are given in the second column of Table 1.

The M/EI diagram for span 1 is represented by two triangles whose areas are $1/2 \times 60 \times \frac{(-7.47)}{1000} = -0.224$ and $1/2 \times 60 \times \frac{14.94}{1000} = +0.448$. Other areas are found in similar fashion and the areas applied to the conjugate beam as loads as shown in Fig. 3. The reaction at support 3 is found from the fact that the moment must be zero in the conjugate beam at the hinge placed at support 2. The other shears are readily found by applying $\Sigma V = 0$ to the various spans considered as free bodies.

The moments in span 1 and for the left portion of span 2 of the conjugate beam will now be found. For convenience a loading of 1000 M/EI has been used. The equations for these moments are also the equations for the same portions of the influence line for moment at point P. The free bodies shown in Figs. 4a and 4b are used in finding moments at any point distant x from the left end of the free body. The trapezoidal M/EI diagram of Fig. 4b was treated as a positive rectangular area of 8.96x plus a negative triangular area of $230\left(\frac{x}{100}\right)^2$

The moments which are the influence ordinates for the real beam in span 1 are:

$$1000y = -7.47\,\frac{x^2}{2} + 224\left(\frac{x}{60}\right)^2\left(\frac{x}{3}\right) + 448\left(\frac{x}{6}\right)^2\frac{x}{3}$$

$$= -3.73\,x^2 + 13440\left(\frac{x}{60}\right)^2$$

For the portion of span 2 left of point P

$$1000y = 224\,x + 8.96\,\frac{x^2}{2} - 230\left(\frac{x}{100}\right)^2\frac{x}{3}$$

Similar moment equations were found for the other spans. They were found to be:

Span 1	$y = \qquad - 13.43\,(x/60)^2 + 13.45\,(x/60)^3$
Span 2 Left	$y = 0.224x + 44.8\,(x/100)^2 - 7.66\,(x/100)^3$
Span 2 Right	$y = 0.110x + 21.8\,(x/100)^2 + 7.66\,(x/100)^3$
Span 3	$y = -0.0247x - 2.34\,(x/50)^2 + 3.80\,(x/50)^3$
Span 4	$y = 0.0.27x \qquad - 0.493\,(x/40)^3$

The influence line for moment at point P of the continuous beam is shown by the solid line in Fig. 5a.

Shear Influence Lines for Beam on Supports

The influence line for shear at P is the deflection curve for the beam when it has been cut and the cut ends given a relative vertical displacement of one while the slope of the beam to the right of P is equal to the slope to the left of P. The real beam has a constant slope and a sudden change in deflection in the region of P. The conjugate beam (Fig. 1x) must consequently be acted on by a couple at P which will produce a sudden change in moment at that point without any change in shear.

The fixation moments for a beam subjected to a unit relative deflection at point P located kL from the left end of the beam are shown in the appendix to be:

$$F_{12} = -6\ EK \qquad F_{21} = +6\ EK$$

It is of importance that these fixation moments are independent of the position of the point P in its span. Because the fixation moments remain constant while P is moved the restraining moments at the points of support of the real beam must be the same for all positions of P within a given span. This means that the portion of the shear influence line outside of the span containing the point P does not vary with changes in the position of P within that span.

After finding the fixation moments the restraining moments at each support are found by moment distribution. M and M/EI diagrams are then drawn, the conjugate beam set up as a series of free bodies carrying an M/EI loading and the influence line for shear found from the equations for moment in each span of the conjugate beam. This work is not shown in detail as it is quite similar to that done in the moment influence line problem. The moments at the supports and M/EI values found were:

Support	Moment	Point	1000(M/EI)
0	+0.012 EK	a	+0.20
1	-0.0243 EK	b	-0.405
2	+0.0246 EK	c	-0.243
3	-0.0067 EK	d	+0.246
4	0	e	+0.410
		f	-0.111
		g	-0.111

The influence line equations were found to be:

Span 1	$y = 0.000360\ (x/60)^2 - 0.000363\ (x/60)^3$
Span 2 left	$y = -0.00615x - 1.21\ (x/100)^2 + 0.815\ (x/100)^3$
Span 2 right	$y = 0.00600x + 1.23\ (x/100)^2 - 0.815\ (x/100)^3$
Span 3	$y = -0.00148x - 0.138\ (x/50)^2 + 0.217\ (x/50)^3$
Span 4	$y = 0.000744x \qquad - 0.0296\ (x/40)^3$

The influence line for shear at P is shown in Fig. 5b. The dotted portion of the curve is the envelope for all shear influence lines for points in span 2.

Effect of Supporting Beam on Columns

If the simple reactions at 1, 2 and 3 are replaced by columns which are attached to the girder with a moment connection the effect of these columns will be to slightly reduce the maximum live load moment in span 2. The influence line for moment was found at x = 40 in span 2 for a beam similar to the one previously studied but supported by 20 ft. columns at points 1, 2 and 3. The columns ($I = 1600''^4$) were rigidly attached to the beam and were assumed to be fixed at the foundations.

That the conjugate for the beam with columns attached is the same as the conjugate beam for the continuous beam on simple supports can be shown by a study of the following considerations. A beam supported on a series of columns in which side-sway is prevented could be replaced by a beam on a series of supports which carries the same loading as the beam on columns and in addition is loaded with moments and shears at the supports equal to those existing at the tops of the columns. The shears produce direct stresses in the beams. The moments produce an M/EI diagram which can be added to that caused by the loads applied to the continuous beam. The total deflection of the beam supported on columns can thus be found by applying the combined M/EI loading to the conjugate of the beam on supports.

End moments for each of the spans were found by moment distribution. The resulting M/EI values are given in Table 1 where they are compared with corresponding values for the beam without columns. The M/EI diagrams were then used as loadings on the conjugate beam and the influence equations obtained for each span. The effect of columns is shown graphically in Fig. 5a.

The data in Table 2 shows that the effect of the column has been to reduce the maximum moment 6 per cent. In a building there would be columns above and below the beam and the columns would be relatively stiffer than those used here so that the reduction in maximum moment would be greater.

Influence Equations for Three Span Beam

The moment and shear influence lines for a continuous beam of three spans can be obtained in terms of the lengths and moments of inertia of the different spans. The resulting equations have the advantage that they can be evaluated by any computer. While the conjugate beam method was used in deriving these equations the approach was somewhat different from that used in the preceeding four span problem. The beam here used has three spans of different lengths. The moment of inertia is constant within each span but may vary from span to span.

The method used in deriving the moment influence line equation will be briefly described. The conjugate beam with its M/EI loading is shown in Fig. 6b. The resisting moments at M_1 and M_2 (Fig. 6a) caused by Mp which is just large enough to make the relative rotation of the ends of the beam which has been cut at P equal to one radian, have both been assumed to be positive. The deflection of point P in the real beam is equal to the moment at P in the conjugate. The moment at P in the conjugate beam using section 1-P as the free body must be equal to the moment at P using section 2-P as the free body. Equating these moments give the relation between M_1 and M_2. The second equation needed to find the values of M_1 and M_2 is based on the fact that the change in slope at P in the real beam is one radian and consequently the change in shear or reaction at P in the conjugate beam must also be equal to one. Solving these two equations gives values of M_1 and M_2 in terms of n and the I and L of each of the three spans. The moments in each of the spans of the conjugate beams were then found in terms of M_1 and M_2. Substituting values of M_1 and M_2 resulted in the final influence line equations which are given in Table 3.

The equations for the shear influence lines were derived by following a procedure similar to that used for the moment influence lines but using a linear relative deflection of unity for the cut ends of the beam at P in place of the unit angular deflection used for moment. The coefficients a, b, c, d given for the shear influence line equations in Table 3 are seen to be independent

of n. Thus equation 11 gives the positive value of shear for any point in span 2 and equation 10 the negative value. Influence lines for shear for any point in span 2 can be drawn within a dotted envelope similar to that shown in Fig. 5b.

The following examples will be used to show the application of these equations to a simple case. A continuous beam of constant moment of inertia has three 10 ft. spans. The supports are numbered 0 to 3 from left to right. Find the moment at point P which is 4 ft. to the right of support 1:

a) due to an unit load in the middle of span 1
b) due to an unit load 2 ft. to the right of support 2

$$n = 0.40 \quad L = 10 \quad A = \frac{2(0.60) + 3(0.60)^2 - (0.60)^3 - (0.40)^3}{2(0.40) + 3(0.40)^2 - (0.40)^3 - (0.60)^3} = 2.00$$

$$KF = 2 + 1 + 4.5 = 7.5$$

a) From Eq. 1 $\quad y = \frac{-5}{7.5} + \left(\frac{5}{10}\right)^3 \left(\frac{10}{7.5}\right) = -0.500$

b) From Eq. 2 $\quad y = \frac{4}{7.5} + \frac{30}{7.5}\left(\frac{2}{10}\right)^2 - \frac{10}{7.5}\left(\frac{1}{2}\right)\left(\frac{2}{10}\right)^3 = +0.688$

Beam as in above example. Find the moment at P due to an unit load placed at that point which is located 4 ft. to the right of support 2.

$$n = 0.40 \quad L = 10 \quad G = \frac{1}{2+2} \quad KH = \frac{3 - \frac{1}{2} - \frac{3}{4} + 2}{0.60} = 6.25$$

From Eq. 8

$$y = 6\left[-\left(\frac{5}{25}\right)\left(\frac{4}{6}\right) + \frac{3}{6.25}\left(\frac{4}{6}\right) + \frac{1}{6.25}\left(\frac{0.2}{0.6}\right)\right] + \left(\frac{6}{10}\right)^3\left(\frac{10}{6.25}\right) = 1.786$$

TABLE 1

Moment and M/EI Values

Point	1000M	I	1000 M/EI Beam on Supports	1000 M/EI Beam on Columns
a	-448EK	15,000	-7.47	-6.30
b	+896EK	15,000	+14.94	+12.60
c	+896EK	25,000	+8.96	+10.00
d	+436EK	25,000	+4.36	+4.52
e	+436EK	15,000	+7.27	+5.80
f	-112EK	15,000	-1.87	-1.70
g	-112EK	15,000	-1.87	-1.32

At point a $1000\ M/EI = -\frac{448}{60}$

TABLE 2

Maximum Ordinates of Influence Line for Moment at Point P (x = 40') in span 2.

Span	Beam on Supports	Beam on Columns
1	-1.96	-1.67
2	+16.0	+15.1
3	-0.76	-0.62
4	+0.19	-0.62

TABLE 3

SPAN	INFLUENCE LINE FOR MOMENT AT P IN SPAN 2	EQ.
0-1	$y = \frac{-x}{K_1 F} + \left(\frac{x}{L_1}\right)^3 \frac{L_1}{K_1 F}$	1
1-P	$y = \frac{2x}{K_1 F} + 3\left(\frac{x}{L_2}\right)^2 \frac{L_2}{K_2 F} + \left(\frac{x}{L_2}\right)^3 \left(\frac{1-A}{A}\right)\left(\frac{L_2}{K_2 F}\right)$	2
2-P	$y = \frac{2x}{K_2 AF} + 3\left(\frac{x}{L_2}\right)^2 \frac{L_2}{K_2 AF} - \left(\frac{x}{L_2}\right)^3 \left(\frac{1-A}{A}\right)\left(\frac{L_2}{K_2 F}\right)$	3
3-2	$y = \frac{-x}{K_3 AF} + \left(\frac{x}{L_3}\right)^3 \frac{L_3}{K_3 AF}$	4

$$K = \frac{I}{L} \qquad A = \left[\frac{\frac{2(1-n)}{K_3} + \frac{2-3n}{K_2}}{\frac{2n}{K_1} + \frac{3n-1}{K_2}}\right] \qquad F = \frac{2}{K_1} + \frac{2}{AK_3} + \frac{3(1+A)}{AK_2}$$

Note that the distance x in each span is measured from the end of that span furthest from the point P

SPAN	INFLUENCE LINE FOR MOMENT AT P IN SPAN 3	EQ.
0-1	$y = x\left(\frac{G}{K_1 H}\right) - \left(\frac{x}{L_1}\right)^3 \left(\frac{G L_1}{K_1 H}\right)$	5
1-2	$y = -x\left(\frac{2G}{K_1 H}\right) - \left(\frac{x}{L_2}\right)^2 \left(\frac{3 G L_2}{K_2 H}\right) + \left(\frac{x}{L_2}\right)^3 \left(\frac{1+G}{K_2 H}\right) L_2$	6
2-P	$y = x\left[\frac{3}{K_2 H} - \frac{G}{H}\left(\frac{2}{K_1} + \frac{3}{K_2}\right)\right] + \left(\frac{x}{L_3}\right)^2 \left(\frac{3 L_3}{K_3 H}\right) - \left(\frac{x}{L_3}\right)^3 \left(\frac{L_3}{K_3 H}\right)$	7
3-P	$y = x\left[\frac{3}{K_2 H}\left(\frac{n}{1-n}\right) - \frac{G}{H}\left(\frac{2}{K_1} + \frac{3}{K_2}\right)\left(\frac{n}{1-n}\right) + \frac{1}{K_3 H}\left(\frac{3n-1}{1-n}\right)\right] + \left(\frac{x}{L_3}\right)^3 \left(\frac{L_3}{K_3 H}\right)$	8

$$K = \frac{I}{L} \qquad G = \frac{K_1}{2K_1 + 2K_2} \qquad H = \frac{\frac{3}{K_2} - \frac{2G}{K_1} - \frac{3G}{K_2} + \frac{2}{K_3}}{1-n}$$

SPAN	INFLUENCE LINE FOR SHEAR AT POINT P IN SPAN 2	EQU.
0-1	$y = \frac{aL_1 x}{bK_1 L_1 L_2} - \frac{aL_1}{bK_1 L_2}\left(\frac{x}{L_1}\right)^3$	9
1-P	$y = -\frac{2x}{bK_1 L_2} - \frac{3a}{bK_2}\left(\frac{x}{L_2}\right)^2 + \frac{(1+a)}{bK_2}\left(\frac{x}{L_2}\right)^3$	10
2-P	$y = \frac{2x}{bK_1 L_2} + \frac{3}{bK_2}\left(\frac{x}{L_2}\right)^2 - \frac{(1+a)}{bK_1}\left(\frac{x}{L_2}\right)^3$	11
2-3	$y = \frac{L_3}{bK_3 L_2}\left(\frac{x}{L_3}\right) + \frac{L_3}{bK_3 L_2}\left(\frac{x}{L_3}\right)^3$	12

$$K = \frac{I}{L} \qquad a = \frac{\frac{3}{K_2} + \frac{3}{K_3}}{\frac{2}{K_1} + \frac{3}{K_2}} \qquad b = \frac{2-a}{K_2} + \frac{2}{K_3}$$

Note that the distance x in each span is measured from the end of that span furthest from the point P.

SPAN	INFLUENCE LINE FOR SHEAR AT POINT P IN SPAN 3	EQU.
1	$y = \frac{cL_1}{dK_1 L_3}\left(\frac{x}{L_1}\right) - \frac{cL_1}{dK_1 L_3}\left(\frac{x}{L_1}\right)^3$	13
2	$y = -\frac{2cL_2}{dK_2 L_3}\left(\frac{x}{L_2}\right) - \frac{3cL_2}{dK_2 L_3}\left(\frac{x}{L_2}\right)^2 + \frac{(1+c)L_2}{dK_2 L_3}\left(\frac{x}{L_2}\right)^3$	14
2-P	$y = \left[-\frac{2c}{dK_1} + \frac{3(1-c)}{dK_2}\right]\frac{x}{L_3} + \frac{3}{dK_3}\left(\frac{x}{L_3}\right)^2 - \frac{1}{dK_3}\left(\frac{x}{L_3}\right)^3$	15
3-P	$y = \left[\frac{2c}{dK_1} - \frac{3(1-c)}{dK_2} - \frac{3}{dK_3}\right]\frac{x}{L_3} + \frac{1}{dK_3}\left(\frac{x}{L_3}\right)^3$	16

$$K = \frac{I}{L} \qquad c = \frac{K_1}{2K_1 + 2K_2} \qquad d = \left[\frac{2c}{K_1} + \frac{3(c-1)}{K_2} - \frac{3}{K_3}\right]$$

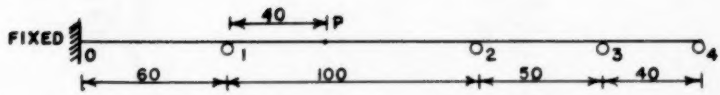
K	K_2	K_3	a	$b\,(K)$	c	$d\,(K)$
2.5K	K	K	1.316	2.684	0.357	-3.64
	1.25K		1.375	2.500	.333	-3.33
	1.5K		1.430	2.380	.313	-3.12
	1.75K		1.478	2.298	.294	-2.98
	2K		1.521	2.240	.278	-2.86
	2.5K		1.600	2.160	.250	-2.70
	3K		1.669	2.110	.227	-2.59
2.5K	K	1.25K	1.210	2.390	0.357	-3.24
	1.25K		1.250	2.200	.333	-2.93
	1.5K		1.286	2.076	.313	-2.72
	1.75K		1.319	1.989	.294	-2.58
	2K		1.348	1.926	.278	-2.46
	2.5K		1.400	1.840	.250	-2.30
	3K		1.444	1.785	.227	-2.19
2.5K	K	1.5K	1.140	2.193	0.357	-2.98
	1.25K		1.168	1.998	.333	-2.67
	1.5K		1.190	1.873	.313	-2.46
	1.75K		1.212	1.783	.294	-2.30
	2K		1.232	1.717	.278	-2.19
	2.5K		1.268	1.636	.250	-2.03
	3K		1.298	1.567	.227	-1.92
2.5K	K	1.75K	1.091	2.053	0.357	-2.79
	1.25K		1.107	1.857	.333	-2.48
	1.5K		1.122	1.738	.313	-2.27
	1.75K		1.136	1.636	.294	-2.11
	2K		1.148	1.569	.278	-2.00
	2.5K		1.171	1.474	.250	-1.84
	3K		1.191	1.412	.227	-1.73

K_1	K_2	K_3	a	$b\,(K)$	c	$d\,(K)$
2.5K	K	2K	1.052	1.948	0.357	-2.64
	1.25K		1.062	1.750	.333	-2.33
	1.5K		1.071	1.619	.313	-2.12
	1.75K		1.080	1.526	.294	-1.98
	2K		1.087	1.456	.278	-1.86
	2.5K		1.100	1.360	.250	-1.70
	3K		1.111	1.296	.227	-1.59
2.5K	K	2.5K	1.000	1.800	0.357	-2.44
	1.25K		1.000	1.600	.333	-2.14
	1.5K		1.000	1.467	.313	-1.92
	1.75K		1.000	1.372	.294	-1.78
	2K		1.000	1.300	.278	-1.66
	2.5K		1.000	1.200	.250	-1.50
	3K		1.000	1.133	.227	-1.39
2.5K	K	3K	.965	1.702	0.357	-2.31
	1.25K		.958	1.500	.333	-2.00
	1.5K		.952	1.365	.313	-1.79
	1.75K		.947	1.269	.294	-1.64
	2K		.942	1.196	.278	-1.53
	2.5K		.933	1.094	.250	-1.37
	3K		.926	1.025	.227	-1.26

$$a = \frac{K_1\left(2K_2 + 3K_3\right)}{K_3\left(3K_1 + 2K_2\right)} \qquad b = \frac{2-a}{K_2} + \frac{2}{K_3}$$

$$c = \frac{K_1}{3K_1 + 2K_2} \qquad d = \left[\frac{2c}{K_1} + \frac{3(c-1)}{K_2} - \frac{2}{K_3}\right]$$

Values of the constants a, b, c, for K_1 = 2.5K and K_2 and K_3 each ranging from K to 3K are presented herewith. The use of these values will facilitate the evaluation of the shear influence line equations given in the second half of Table 3.



I INCHES	15000	25000	15000	15000
K IN^4/FT	250	250	300	375
RELATIVE K	K	K	1.20 K	1.500 K

FIG 1A THE REAL BEAM

FIG 1B CONJUGATE BEAM FOR MOMENT INFLUENCE LINE

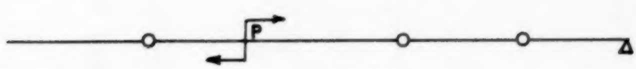


FIG 1C

CONJUGATE BEAM FOR SHEAR INFLUENCE LINE

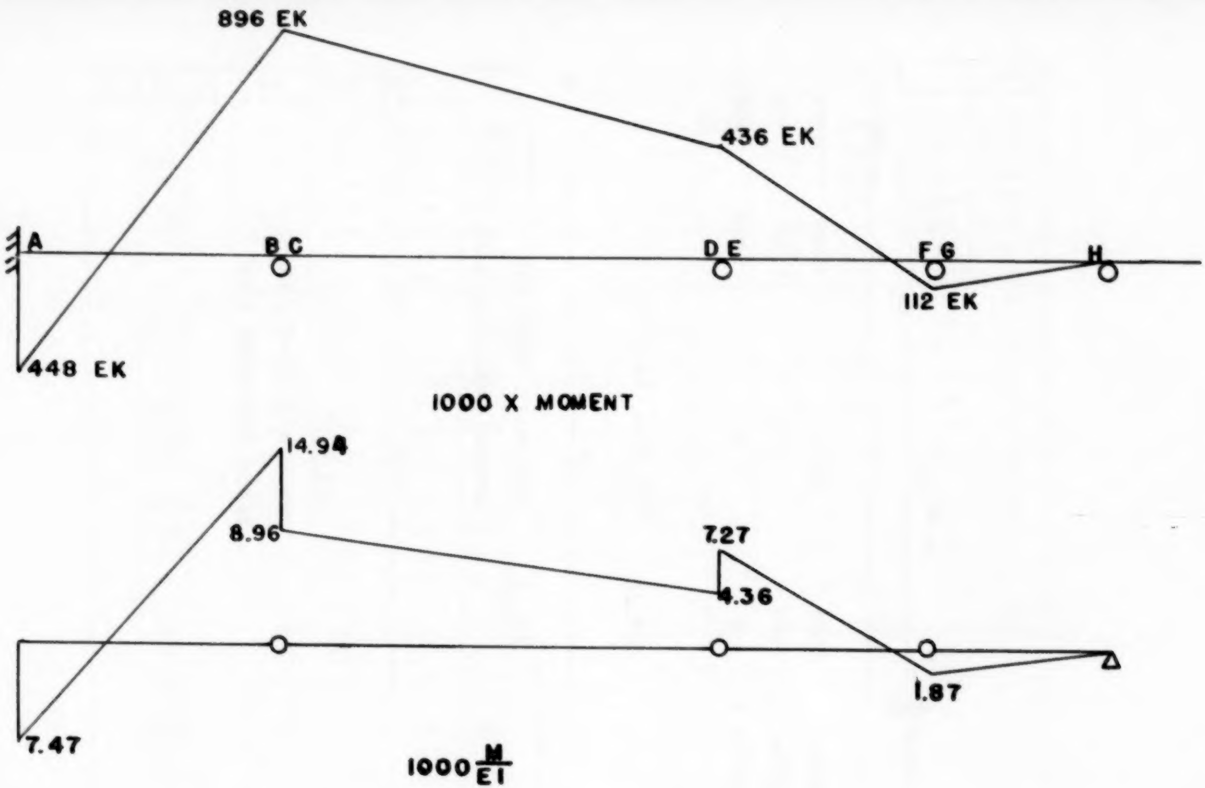


FIG 2 MOMENT AND $\frac{M}{EI}$ DIAGRAMS

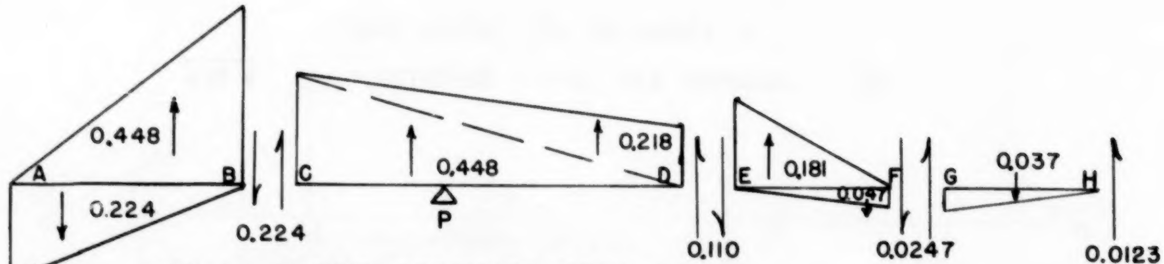
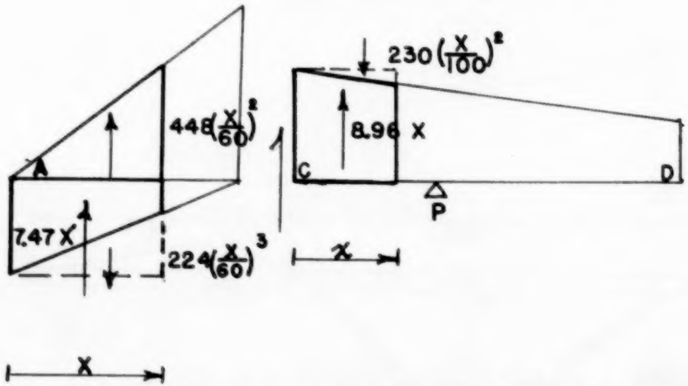


FIG 3 LOADED CONJUGATE BEAM AS A SERIES OF FREE BODIES



$$\left(\frac{8.96 - 4.36}{2}\right) 100 = 230$$

FIG 4 PORTIONS OF SPANS 1 AND 2 TREATED AS FREE BODIES

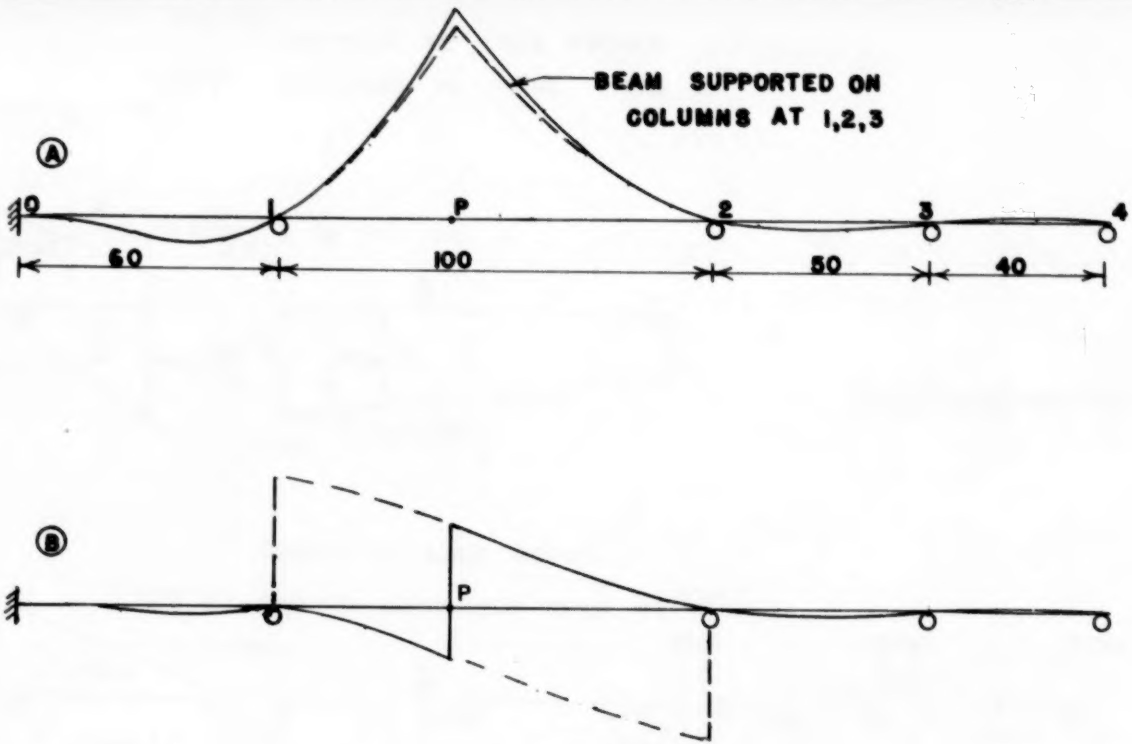


FIG 5 INFLUENCE LINES FOR MOMENT (A) AND SHEAR (B) AT POINT P

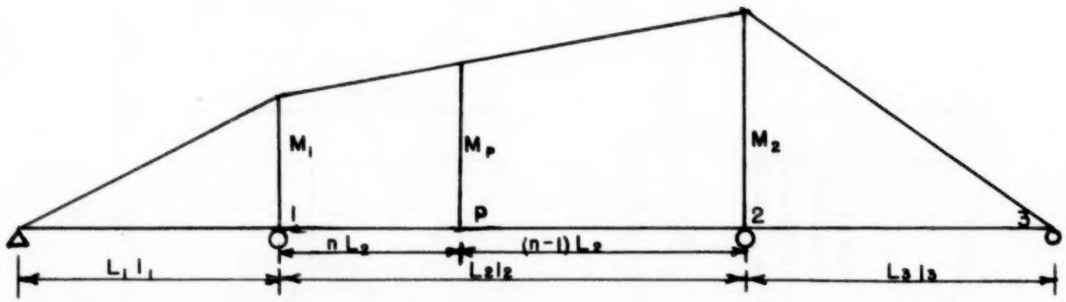


FIG 6A MOMENT DIAGRAM DUE TO

M_p ACTING AT P

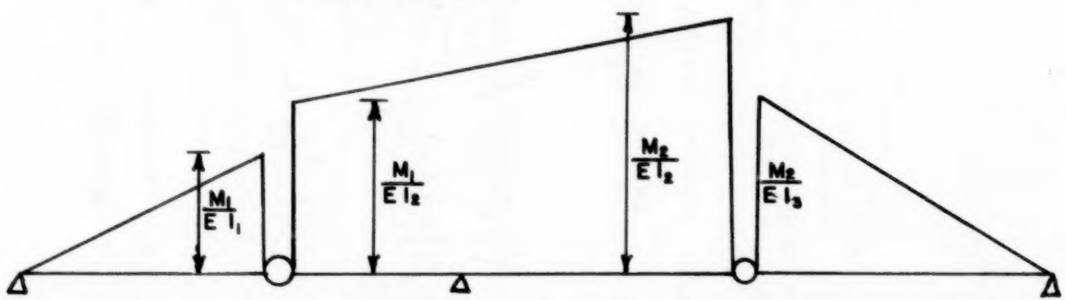


FIG 6 B CONJUGATE BEAM LOADED

WITH $\frac{M}{EI}$ DIAGRAM

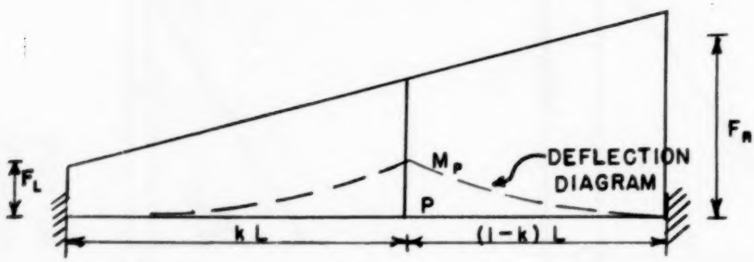


FIG 7a MOMENT DIAGRAM DUE TO M_p

CHANGE IN SLOPE AT P = 1 RADIAN

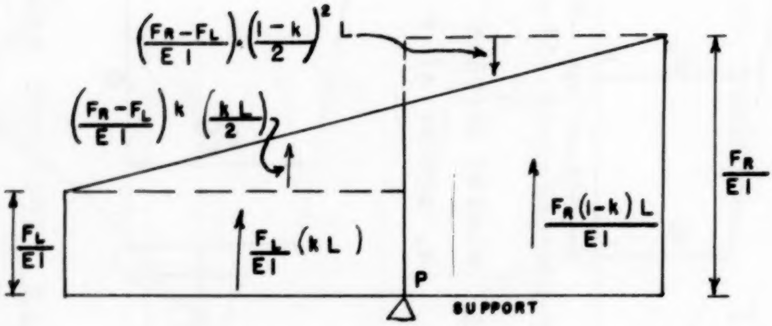


FIG 7b CONJUGATE BEAM

WITH $\frac{M}{EI}$ LOADING

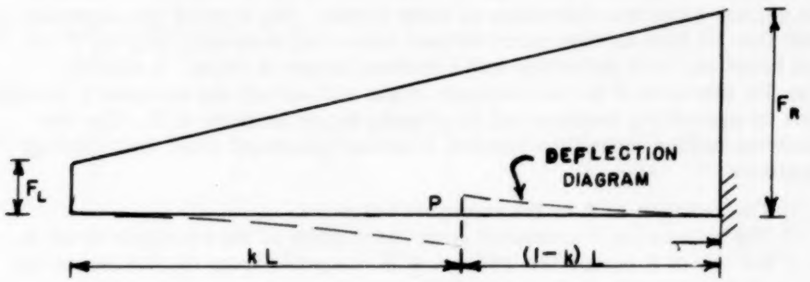


FIG 8a MOMENT DIAGRAM

CHANGE IN DEFLECTION AT P = 1

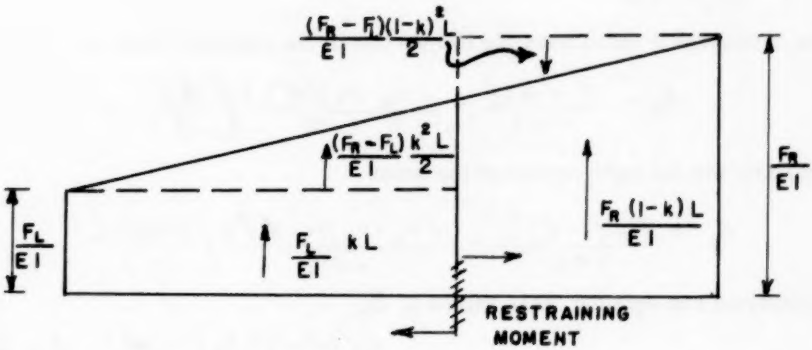


FIG 8b CONJUGATE BEAM

WITH $\frac{M}{EI}$ LOADING

APPENDIX

Fixation Moments Due to Unit Angular Deflection at P.

The conjugate beam method will be used to find the fixation moments F_L and F_R at the ends of a beam which has been hinged at point P and is acted on by moments Mp which cause a relative angular deflection of one radian in the region of P as shown in Fig. 7a. The real beam is fixed at the ends so has neither slope nor deflection at those points. The ends of the conjugate must then be free as they carry neither shear nor moment. At point P the real beam has both deflection and a sudden change of slope. A support (Fig. 7b) placed at P in the conjugate beam will satisfy the necessary conditions by permitting moment and an abrupt change in shear at P. The two equations needed to find the fixation moments are found from the following conditions:

1) The reaction at P in the conjugate beam = 1
2) The moment at P computed from the portion of the conjugate beam to the left of P equals the moment at P computed from the portion of the conjugate beam to the right of P.

From $\Sigma V = 0$ the reaction at P is equal to the area of the $\frac{M}{EI}$ diagram. Consequently:

$$\left(\frac{F_L + F_R}{EI}\right)\frac{L}{2} = 1 \qquad\qquad F_L + F_R = \frac{2EI}{L} \qquad \text{Eq. 1}$$

The moment at P found from the left portion of the conjugate beam is:

$$\delta_p = \frac{F_L (kL)^2}{2EI} + \frac{(F_R - F_L)(k^2 L)}{2EI}\left(\frac{kL}{3}\right)$$

Similarly for the right portion of the beam

$$\delta_p = \frac{F_R (1-k)^2 L^2}{2EI} - \frac{(F_R - F_L)(1-k)^2 L}{2EI}\left(\frac{1-k}{3}\right)L$$

Simplifying and equating these values of δ_p

$$F_L(3k-1) = F_R(2-3k) \qquad \text{Eq. 2}$$

Solving equations 1 and 2 simultaneously

$$\underline{F_R = \frac{EI}{L}(6k-2)} \qquad\qquad \underline{F_L = \frac{EI}{L}(4-6k)}$$

Fixation Moments Due to Linear Deflection at P

The fixation moments F_L and F_R for a beam of constant moment of inertia which has been cut at P and the cut ends forced apart so that they have a relative vertical deflection of one while maintaining the same slope either side of P (Fig. 8a.) can be found by use of the beam's conjugate. The ends of the conjugate beam will be free as the real beam has fixed ends. At P in the real beam there is a sudden change in deflection but no change in slope. This calls for a conjugate beam so supported that there will be a sudden change in mo-

ment but no change in shear. This condition is satisfied by placing a resisting couple at P as shown in Fig. 8b in which both fixation are assumed to be positive. The two equations needed to find the fixation moments are found from the conditions:

1) Slope at both ends of the beam = 0
2) Deflection to right of P - deflection to left of P = 1.00

From 1, the shears at the ends of the conjugate beam must = 0

$$0 + \frac{M_L + M_R}{EI}\left(\frac{L}{2}\right) = 0 \qquad M_L = -M_R \quad \text{Eq. 1}$$

From 2. $$\frac{M_R}{EI}(1-k)^2\frac{L^2}{2} - \frac{(M_R - M_L)(1-k)^2 L}{EI}\frac{(1-k)L}{3} - \left[\frac{M_L}{EI}\frac{k^2L^2}{2}\right.$$

$$\left. + (M_R - M_L)k^2\frac{L}{2}\left(\frac{kL}{3}\right)\right] = 1 \qquad \text{Eq. 2}$$

Substituting $M_L = -M_R$ in 2

$$3M_R(1-k)^2 - M_R(1-k)^3 - M_R k^3 - M_R(1-k)^3$$

$$+ 3M_R k^2 - M_R k^3 = \frac{6EK}{L}$$

$$\underline{M_R = F_R = \frac{6EK}{L}}$$

$$\underline{M_L = F_L = -\frac{6EK}{L}}$$

PROCEEDINGS PAPERS

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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JULY: 457(AT), 458(AT), 459(AT)^c, 460(IR), 461(IR), 462(IR), 463(IR)^c, 464(PO), 465(PO)^c.

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)^c, 479(HY)^c, 480(ST)^c, 481(SA)^c, 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)^c, 488(ST)^c, 489(HY), 490(HY), 491(HY)^c, 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)^c, 502(WW), 503(WW), 504(WW)^c, 505(CO), 506(CO)^c, 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)^c, 519(IR), 520(IR), 521(IR), 522(IR)^c, 523(AT)^c, 524(SU), 525(SU)^c, 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)^c, 531(EM), 532(EM)^c, 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)^c, 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)^c, 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)^c, 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)^c, 569(SM), 570(SM), 571(SM), 572(SM)^c, 573(SM)^c, 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)^c, 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)^c, 596(HW), 597(HW), 598(HW)^c, 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)^c, 607(EM).

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MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)^c, 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)^c, 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)^c, 655(SA), 656(SM)^c, 657(SM)^c, 658(SM)^c.

APRIL: 659(ST), 660(ST), 661(ST)^c, 662(ST), 663(ST), 664(ST)^c, 665(HY)^c, 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

MAY: 679(ST), 680(ST), 681(ST), 682(ST)^c, 683(ST), 684(ST), 685(SA), 686(SA), 687(SA), 688(SA), 689(SA)^c, 690(EM), 691(EM), 692(EM), 693(EM), 694(EM), 695(EM), 696(PO), 697(PO), 698(SA), 699(PO)^c, 700(PO), 701(ST)^c.

JUNE: 702(HW), 703(HW), 704(HW)^c, 705(IR), 706(IR), 707(IR), 708(IR), 709(HY)^c, 710(CP), 711(CP), 712(CP), 713(CP)^c, 714(HY), 715(HY), 716(HY), 717(HY), 718(SM)^c, 719(HY)^c, 720(AT), 721(AT), 722(SU), 723(WW), 724(WW), 725(WW), 726(WW)^c, 727(WW), 728(IR), 729(IR), 730(SU)^c, 731(SU).

JULY: 732(ST), 733(ST), 734(ST), 735(ST), 736(ST), 737(PO), 738(PO), 739(PO), 740(PO), 741(PO), 742(PO), 743(HY), 744(HY), 745(HY), 746(HY), 747(HY), 748(HY)^c, 749(SA), 750(SA), 751(SA), 752(SA)^c, 753(SM), 754(SM), 755(SM), 756(SM), 757(SM), 758(CO)^c, 759(SM)^c, 760(WW)^c.

c. Discussion of several papers, grouped by Divisions.

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